International Journal of Computer Discovered Mathematics (IJCDM)
ISSN 2367-7775 ©IJCDM
Volume 6, 2021, pp. 6–42
Received 6 January 2021. Published on-line 14 February 2021
web: http://www.journal-1.eu/

Computer Investigation of Properties of the Gergonne Point of a Triangle

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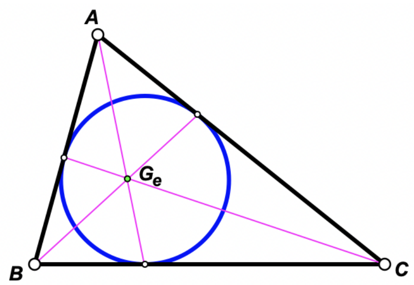
Abstract. The incircle of a triangle touches the sides of the triangle in three points. It is well-known that the lines from these points to the opposite vertices meet at a point known as the Gergonne point of the triangle. We use a computer to discover and catalog properties of the Gergonne point.

Keywords. triangle geometry, Gergonne point, computer-discovered mathematics, Mathematica, GeometricExplorer.

Mathematics Subject Classification (2020). 51M04, 51-08.

1. Introduction

The line from a vertex of a triangle to the contact point of the incircle with the opposite side is called a *Gergonne cevian*. It is well known that the three Gergonne cevians of a triangle meet in a point. This point is known as the Gergonne point of the triangle. It is shown in the following illustration.



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We use a computer to investigate properties of the Gergonne point. Some of the properties found have been discovered before, and we will give a reference, if known, to these results. In other cases, the property is new.

We systematically search for properties of the Gergonne point by starting with a configuration that consists of a triangle and its Gergonne point. Then we apply a sequence of 0 to 2 basic constructions to this configuration. Basic constructions consist of typical ruler and compass constructions such as finding the intersection of a line with a line or a circle, dropping a perpendicular from a point to a line, drawing parallel and perpendicular lines, constructing an incircle or circumcircle, etc. We also included constructions involving triangle centers, such as constructing centroids, orthocenters, and symmedian points of a triangle. Some Apollonius circle constructions, including center and all touch points (PPP, PPL, PPC, LLL, LLP, LLC, CLP) were also included. This includes drawing incircles, excircles, circumcircles, and mixtilinear incircles. All possible sequences of such constructions are generated.

Notation for Constructions	
Notation	**Description**
$\triangle ABC$	Triangle ABC
a, b, c	The lengths of the sides of $\triangle ABC$
s	$(a+b+c)/2$
$\odot ABC$	The circle through points A, B, and C
$r(ABC)$	The radius of the incircle of $\triangle ABC$
$R(ABC)$	The radius of the circumcircle of $\triangle ABC$
$[F]$	The area of figure F
$\texttt{Gergonne}[ABC]$	The Gergonne point of $\triangle ABC$
$\texttt{Nagel}[ABC]$	The Nagel point of $\triangle ABC$
$\texttt{Spieker}[ABC]$	The Spieker center of $\triangle ABC$
$\texttt{orthocenter}[ABC]$	The orthocenter of $\triangle ABC$
$\texttt{mittenpunkt}[ABC]$	The mittenpunkt of $\triangle ABC$
$\texttt{Feuerbach}[ABC]$	The Feuerbach point of $\triangle ABC$
$\texttt{parallel}[P,AB]$	The line through P parallel to AB
$\texttt{perpendicular}[P,AB]$	The line through P perpendicular to AB
$\texttt{foot}[A,BC]$	The foot of the perpendicular from A to BC
$\texttt{incircle}[ABC]$	The incircle of $\triangle ABC$
$\texttt{incenter}[ABC]$	The incenter of $\triangle ABC$
$\texttt{centroid}[ABC]$	The centroid of $\triangle ABC$
$\texttt{midpoint}[AB]$	The midpoint of line segment AB
$\texttt{ninepointcenter}[ABC]$	The nine-point center of $\triangle ABC$
$\texttt{colline}[A,B,C]$	The points A, B, and C colline.
$\texttt{concur}[AB,CD,EF]$	The lines AB, CD, and EF are concurrent.

After each construction is applied, the resulting figure is analyzed for basic properties, such as three points being collinear, three lines being concurrent, checking for perpendicular or parallel lines, congruent incircles, linear and quadratic relationships between lengths of line segments, etc. Trivial properties are excluded, as well as standard properties of the construction that is applied. The process is carried out by a program called GeometricExplorer that we wrote specifically for this purpose. In some cases, when GeometricExplorer found interesting numerical properties, we used a Mathematica program to find a more general result.

Convention: Gergonne points in our figures will be colored green.

Proofs: The program we used, GeometricExplorer, works by starting with numerical figures and examining the figures constructed using 15 decimal places of accuracy. Although any result found is most likely true, this does not constitute a proof. Using barycentric coordinates and Mathematica, we have found proofs for all of the properties discovered. The proofs are in the Mathematica notebook supplied with the supplementary material associated with this paper.

References: A reference in blue indicates that the result was discovered by our investigation and is believed to be new and was posted to social media in the hopes that someone would find a purely geometric solution. An asterisk after a reference indicates that no one has found a purely geometrical proof as of January 2021.

2. A Triangle and its Gergonne Point

In this section, we report on properties found for a starting figure consisting of a triangle together with its Gergonne point.

2.1. Arbitrary Triangle.

Intrinsic Properties

An *intrinsic property* of a figure is a property of that figure before any constructions are applied. For example, an intrinsic property of an equilateral triangle is that the angles all have measure 60°.

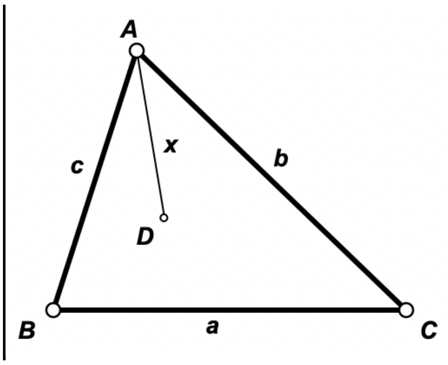
Property 2.1.1. (Spoke Formula)

Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$. $x = DA$.

Conclusion:

$$x = \sqrt{\frac{a(b+c-a)^3\,(a(a+b+c)-2(b-c)^2)}{(a^2+b^2+c^2-2ab-2bc-2ca)^2}}.$$

References: See supplementary notebook.



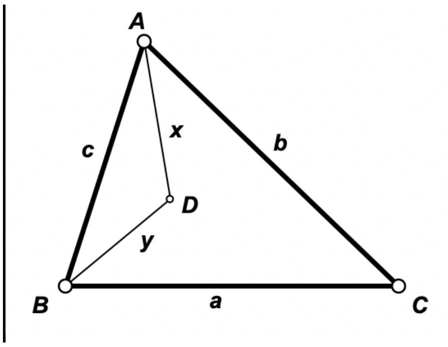
Property 2.1.2. (Tripolar Coordinates)

Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$. $x = DA$, $y = DB$.

Conclusion:

$$\frac{x}{y} = \frac{b+c-a}{c+a-b}\sqrt{\frac{a(b+c-a)(a^2+ab-2b^2+ac+4bc-2c^2)}{b(c+a-b)(b^2+bc-2c^2+ba+4ca-2a^2)}}.$$

References: This follows from the SpokeFormula. See also [12].



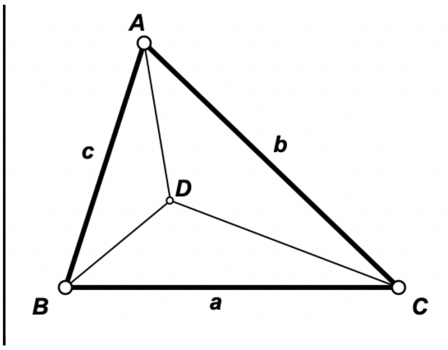
Property 2.1.3. (Barycentric Coordinates)

Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.

Conclusion:

$$\frac{[BDC]}{[CDA]} = \frac{c+a-b}{b+c-a}.$$

References: [12]



Property 2.1.4. (Area Formula)

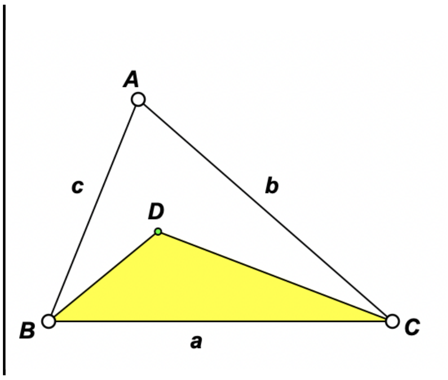
Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.

Conclusion:

$$[BCD] = \frac{(a+b-c)(a-b+c)}{2ab+2bc+2ca-a^2-b^2-c^2}K,$$

where $K = \sqrt{s(s-a)(s-b)(s-c)}$.

References: See supplementary notebook.



1-step Properties

A *1-step property* of a figure is a property found after applying exactly one basic construction to the given figure.

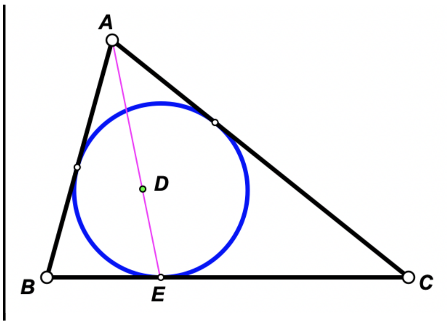
Property 2.1.5. (Defining Property of the Gergonne Point)

Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.

Step 1: $E = \texttt{incircle}[ABC] \cap BC$

Conclusion: $\texttt{colline}[A, D, E]$.

References: [1, p. 160], [10, p. 184]



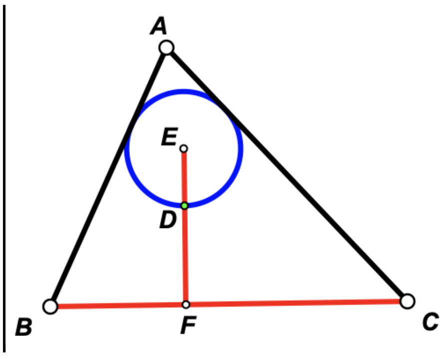
Property 2.1.6. (LLP Property)

Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.

Step 1: A circle, center E, passes through D and is tangent to AB and AC.

Conclusion: $DE \perp BC$.

References: [25]



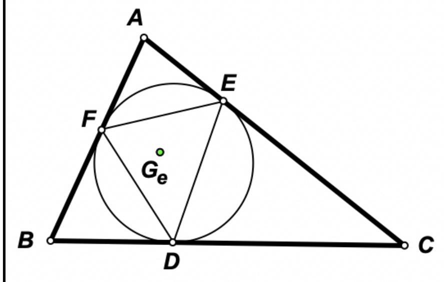
Property 2.1.7. (Intouch Symmedian Point)

Start with: Triangle ABC.
$G_e = \texttt{Gergonne}[ABC]$.

Step 1: The incircle of $\triangle ABC$ touches BC at D, touches AC at E, and touches AB at F.

Conclusion: G_e is the symmedian point of $\triangle DEF$.

References: [12]



Property 2.1.8.

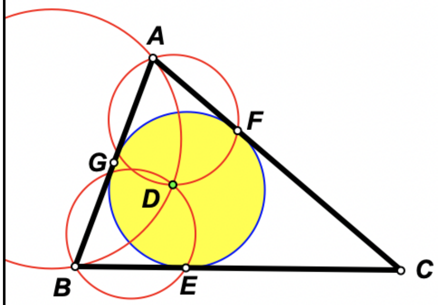
Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.

Step 1: The incircle of $\triangle ABC$ touches BC at E, touches AC at F, and touches AB at G.

Conclusion:

$R(AFD) + R(BED) = R(ABD)$.

References: [48]



2.2. Triangles with Angle Restrictions.

In this section, we report on properties found when the starting triangle has restrictions placed on its angles.

Intrinsic Properties

No intrinsic properties were found.

1-step Properties

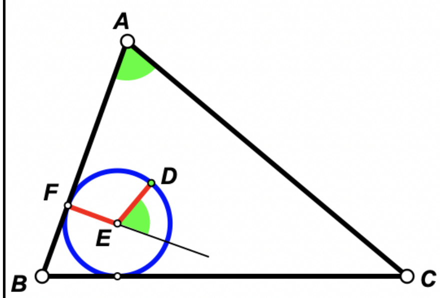
Property 2.2.1.

Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.

Step 1: A circle, center E, passes through D and is tangent to AB at F and tangent to BC.

Conclusion: $\angle DEF + \angle BAC = 180°$.

References: [18]



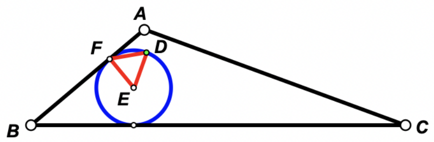
Property 2.2.2.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $\angle A = 120°$.
$D = \texttt{Gergonne}[ABC]$.

Step 1: A circle, center E, passes through D and is tangent to AB at F and tangent to BC.

Conclusion: $\triangle DEF$ is equilateral.

References: [30], special case of previous property



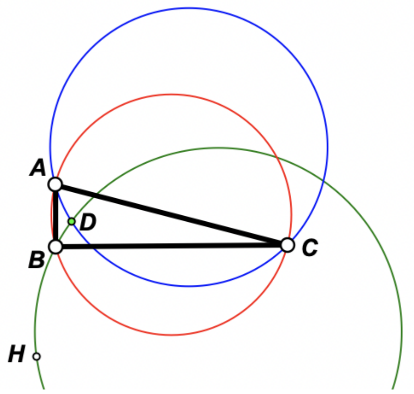
Property 2.2.3.

Start with: Right Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.
$\triangle ABC$ satisfies $\angle A = 75°$ and $\angle C = 15°$.

Step 1: $H = \texttt{orthocenter}[ACD]$.

Conclusion:
$[\odot ABC] + [\odot ACD] = [\odot BDH]$.

References: [29]*



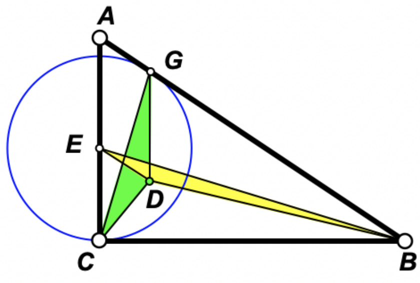
Property 2.2.4.

Start with: Right triangle ACB.
$D = \texttt{Gergonne}[ABC]$.
$\triangle ABC$ satisfies $\angle C = 90°$.

Step 1: E is the center of a circle that touches BC at C and touches AB at G.

Conclusion: $[BDE] = [CDG]$.

References: [31]*



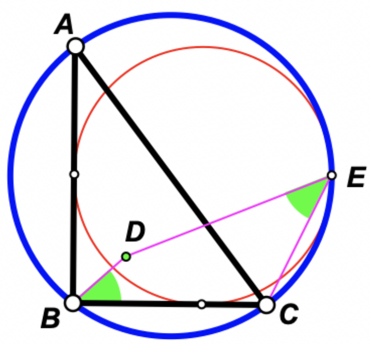
Property 2.2.5.

Start with: Right triangle ABC.
$\triangle ABC$ satisfies $\angle B = 90°$.
$D = \texttt{Gergonne}[ABC]$.

Step 1: The B-mixtilinear incircle touches $\odot ABC$ at E.

Conclusion: $\angle DBC = \angle DEC$.

References: [37]*



2.3. Triangles with Side Restrictions.

In this section, we report on properties found when the starting triangle has restrictions placed on its sides.

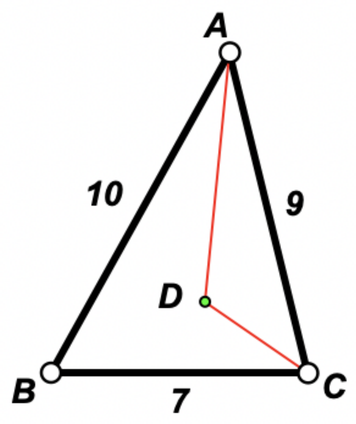
Intrinsic Properties

Property 2.3.1.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 7 : 9 : 10$.
$D = \texttt{Gergonne}[ABC]$.

Conclusion: $AD = 2CD$.

References: This follows from the Tripolar Coordinates.

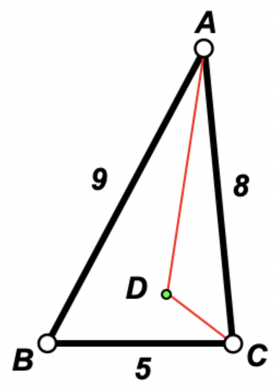


Property 2.3.2.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 5 : 8 : 9$.
$D = \texttt{Gergonne}[ABC]$.

Conclusion: $AD = 3CD$.

References: This follows from the Tripolar Coordinates.

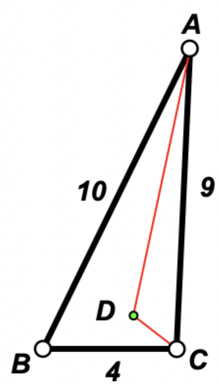


Property 2.3.3.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 4 : 9 : 10$.
$D = \texttt{Gergonne}[ABC]$.

Conclusion: $AD = 5CD$.

References: This follows from the Tripolar Coordinates.

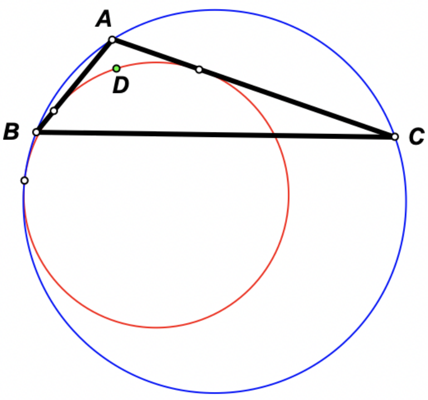


Property 2.3.4.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $as = b^2 + bc + c^2$.
$D = \texttt{Gergonne}[ABC]$.

Conclusion: The A-mixtilinear incircle of $\triangle ABC$ passes through D.

References: [3]



1-step Properties

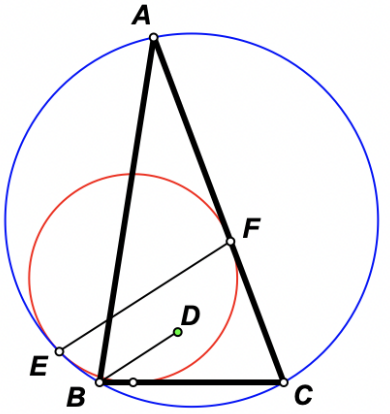
Property 2.3.5.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $b = 2a$.
$D = \texttt{Gergonne}[ABC]$.

Step 1: The C-mixtilinear incircle touches AC at F and touches $\odot ABC$ at E.

Conclusion: $BD \parallel EF$.

References: [32]*



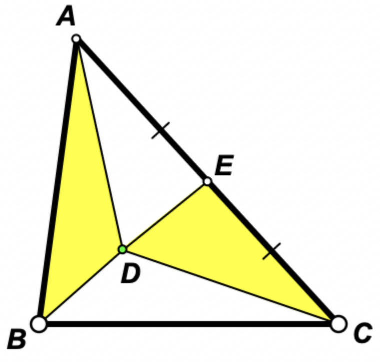
Property 2.3.6.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a = 3(b - c)$.
$D = \texttt{Gergonne}[ABC]$.

Step 1: $E = \texttt{midpoint}[AC]$.

Conclusion: $[ABD] = [CED]$.

References: [34]



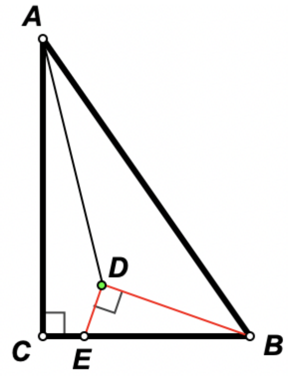
Property 2.3.7.

Start with: Right triangle ABC.
$\triangle ABC$ satisfies $\angle C = 90°$.
$D = \texttt{Gergonne}[ABC]$.

Step 1: $E = \texttt{perpendicular}[D, BD] \cap BC$

Conclusion: $\dfrac{BD}{DE} = \dfrac{2s}{b}$.

References: [79]



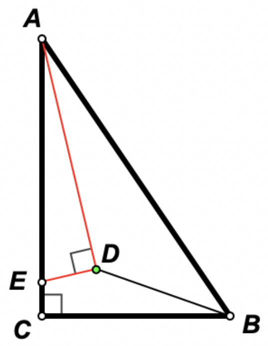
Property 2.3.8.

Start with: Right triangle ABC.
$\triangle ABC$ satisfies $\angle C = 90°$.
$D = \texttt{Gergonne}[ABC]$.

Step 1: $E = \texttt{perpendicular}[D, AD] \cap AC$

Conclusion: $\dfrac{AD}{DE} = \dfrac{2s}{a}$.

References: [79]



Property 2.3.9.

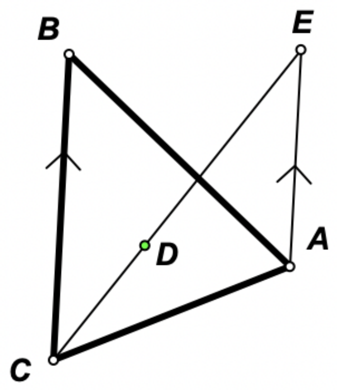
Start with: Isosceles triangle ABC.
$\triangle ABC$ satisfies $a = c$.
$D = \texttt{Gergonne}[ABC]$.

Step 1: $E = \texttt{parallel}[A, BC] \cap CD$.

Conclusion:

$$\frac{1}{BC} + \frac{1}{AE} = \frac{2}{AC}.$$

References: [74]



2.4. **A Triangle with a Gergonne Point and Another Center.**

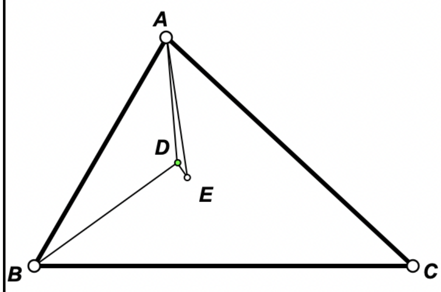
In this section, we report on properties found when including the Gergonne point and another triangle center in the starting triangle.

Property 2.4.1.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $\angle B = 60°$.
$D =$ `Gergonne`$[ABC]$.
$E =$ `incenter`$[ABC]$.

Conclusion: $\angle ABD = \angle AED$.

References: [70]

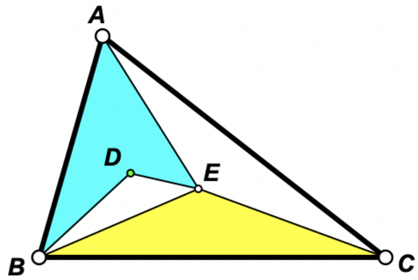


Property 2.4.2.

Start with: Triangle ABC.
$D =$ `Gergonne`$[ABC]$.
$E =$ `mittenpunkt`$[ABC]$.

Conclusion: $[ABDE] = [BCE]$.

References: [69]

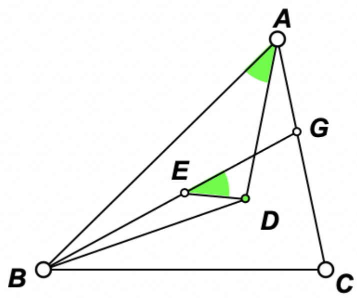


Property 2.4.3.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $2a = b + c$.
$D =$ `Gergonne`$[ABC]$.
$E =$ `Nagel`$[ABC]$.

Conclusion: $\angle BAD = \angle DEG$.

References: [68]

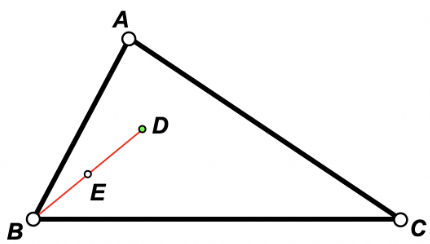


Property 2.4.4.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $2(s - a)(s - c) = b(s - b)$.
$D =$ `Gergonne`$[ABC]$.
$E =$ `Feuerbach`$[ABC]$.

Conclusion: B, E, and D colline and $BE = DE$.

References: [87]

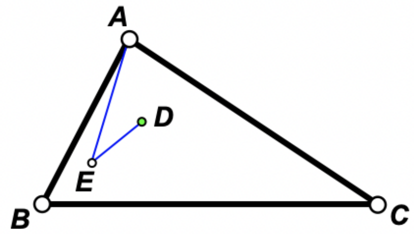


Property 2.4.5.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $(s-b)(s-c) = a(s-a)$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{Feuerbach}[ABC]$.

Conclusion: $AE = 2DE$.

References: [87]

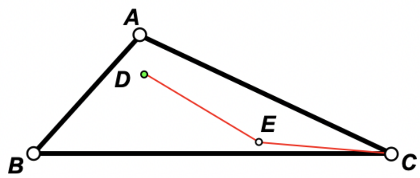


Property 2.4.6.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 9 : 7 : 4$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{Nagel}[ABC]$.

Conclusion: $CE = DE$.

References: [71]

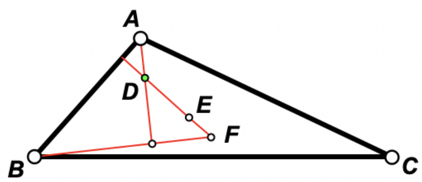


Property 2.4.7.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 9 : 7 : 4$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{centroid}[ABC]$.
$F = \texttt{mittenpunkt}[ABC]$.

Conclusion: $AB \perp DE$ and $AD \perp BF$.

References: [72]



Note: Our investigation found many other perpendicularities involving the Gergonne point and another triangle center in various shape triangles. We omit them from this report since there were so many instances.

3. A Triangle with a Gergonne Cevian

In this section, we report on properties found for a starting figure consisting of a triangle and one of its Gergonne cevians.

3.1. Intrinsic Properties.

Property 3.1.1. (Trace Lengths)

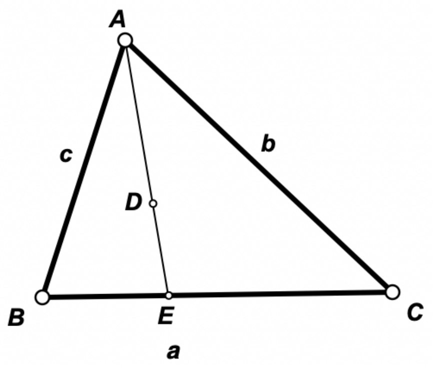
Start with:
Triangle ABC with Gergonne cevian AE.
$D = \texttt{Gergonne}[ABC]$.

Conclusion:

$BE = s - b;\qquad CE = s - c;$

$$\frac{BE}{CE} = \frac{s-b}{s-c}.$$

References: [1, p. 87]

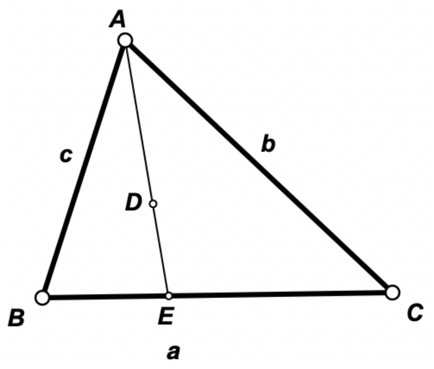


Property 3.1.2. (Cevian Division)

Start with:
Triangle ABC with Gergonne cevian AE.
$D = \texttt{Gergonne}[ABC]$.

Conclusion: $\dfrac{AD}{DE} = \dfrac{a(s-a)}{(s-b)(s-c)}.$

References: [1, p. 164]



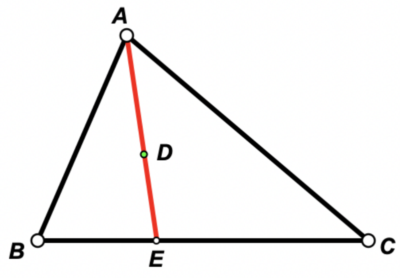
Property 3.1.3. (Cevian Length)

Start with:
Triangle ABC with Gergonne cevian AE.
$D = \texttt{Gergonne}[ABC]$.

Conclusion:

$$AE^2 = \frac{(s-a)(as-(b-c)^2)}{a}.$$

References: See supplementary notebook.

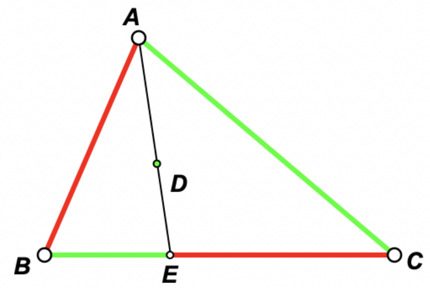


Property 3.1.4. (Split Perimeter Property)

Start with:
Triangle ABC with Gergonne cevian AE.
$D = \texttt{Gergonne}[ABC]$.

Conclusion: $AB + CE = AC + BE = s$.

References: This follows from the trace lengths.

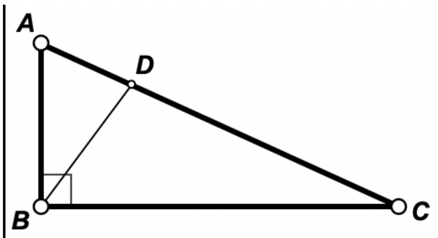


Property 3.1.5.

Start with:
Triangle ABC with Gergonne cevian BD.
$\triangle ABC$ satisfies $\angle B = 90°$.

Conclusion: $AD \cdot CD = [ABC]$.

References: [46]

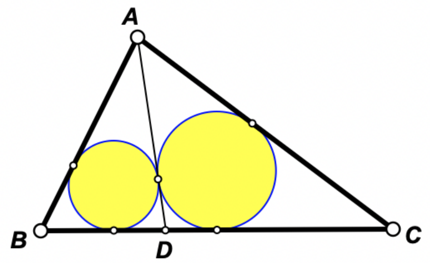


Property 3.1.6. (Kissing Circles Theorem)

Start with:
Triangle ABC with Gergonne cevian AD.

Conclusion:
Incircles of $\triangle ABD$ and $\triangle ADC$ touch.

References: [16]

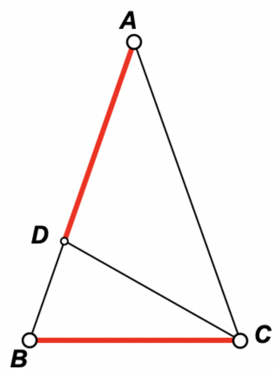


Property 3.1.7.

Start with:
Triangle ABC with Gergonne cevian CD.
$\triangle ABC$ satisfies $3a = b + c$.

Conclusion: $AD = BC$.

References: This follows from the trace lengths.

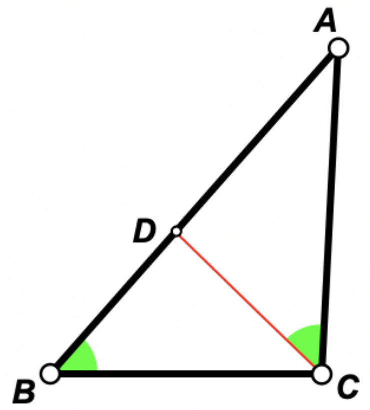


Property 3.1.8.

Start with:
Triangle ABC with Gergonne cevian CD.
$\triangle ABC$ satisfies $b^2 = c(s-a)$.

Conclusion: $\angle ABC = \angle ACD$.

References: [83]



3.2. **1-step Properties.**

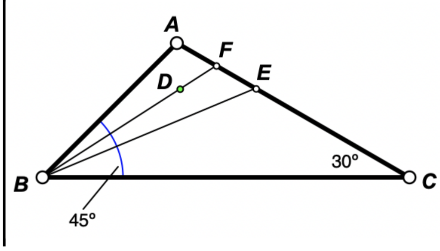
Property 3.2.1.

Start with:
Triangle ABC with Gergonne cevian BF.
$\triangle ABC$ satisfies $\angle B = 45°$ and $\angle C = 30°$.
$D = \texttt{Gergonne}[ABC]$

Step 1: BE bisects $\angle ABC$.

Conclusion: $AF = FE$.

References: [27]*



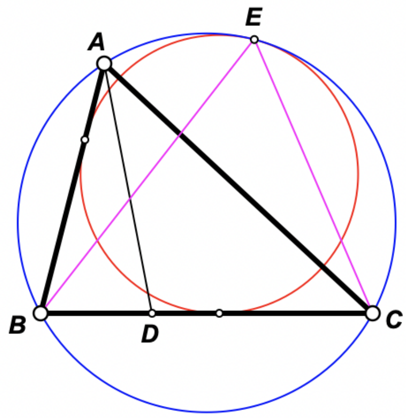
Property 3.2.2.

Start with:
Triangle ABC with Gergonne cevian AD.

Step 1: The B-mixtilinear incircle touches $\odot ABC$ at E.

Conclusion: $AB \cdot CE = BE \cdot CD$.

References: [45]



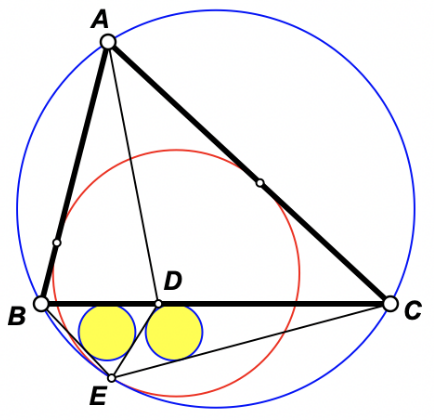
Property 3.2.3.

Start with:
Triangle ABC with Gergonne cevian AD.

Step 1: The A-mixtilinear incircle touches $\odot ABC$ at E.

Conclusion: $r(BED) = r(CDE)$.

References: [23, Theorem 3.4]



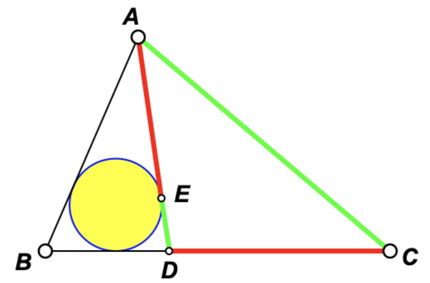
Property 3.2.4.

Start with:
Triangle ABC with Gergonne cevian AD.

Step 1: The incircle of $\triangle ABD$ touches AD at E.

Conclusion: $AC + DE = AE + CD$.

References: This follows from the split perimeter property.



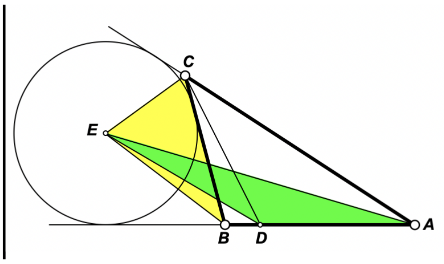
Property 3.2.5.

Start with:
Triangle ABC with Gergonne cevian CD.
$\triangle ABC$ satisfies $b + c = 3a$.

Step 1: E is A-excenter of $\triangle ABC$.

Conclusion: $[AED] = [BCE]$.

References: [19]



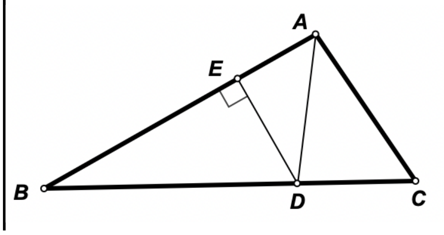
Property 3.2.6.

Start with:
Triangle ABC with Gergonne cevian AD.
$\triangle ABC$ satisfies $\angle C = 2\angle B$.

Step 1: $E = \texttt{perpendicular}[D, AB]$.

Conclusion: $\dfrac{1}{BD} + \dfrac{1}{BE} = \dfrac{1}{CD}$.

References: [51]



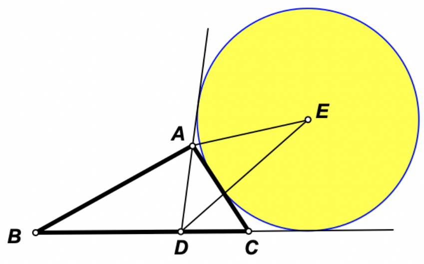
Property 3.2.7.

Start with:
Triangle ABC with Gergonne cevian AD.
$\triangle ABC$ satisfies $\angle C = 2\angle B$.

Step 1: E is the center of the D-excircle of $\triangle ADC$.

Conclusion: $\angle ABC = \angle AED$.

References: [52]



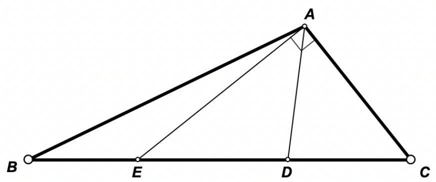
Property 3.2.8.

Start with:
Triangle ABC with Gergonne cevian AD.
$\triangle ABC$ satisfies $\angle B : \angle C : \angle A = 1 : 2 : 4$.

Step 1: $E = \texttt{perpendicular}[AC, A] \cap BC$.

Conclusion: $BD + BE = 3CD$.

References: [53]



3.3. **A Triangle with a Gergonne Cevian and Two Centers.**

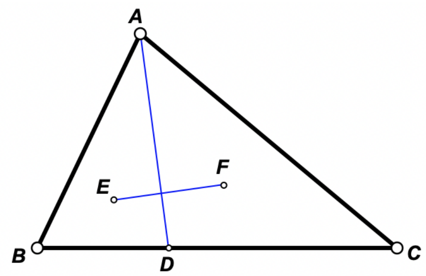
In this section, we report on properties found for a starting figure consisting of a triangle and one of its Gergonne cevians along with a triangle center constructed inside each of the two triangles formed.

Property 3.3.1.

Start with:
Triangle ABC with Gergonne cevian AD.
$E = \texttt{incenter}[ABD]$.
$F = \texttt{incenter}[ADC]$.

Conclusion: $AD \perp EF$.

References: [38], follows from the Kissing Circles Theorem.

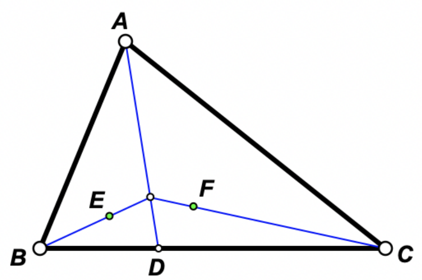


Property 3.3.2.

Start with:
Triangle ABC with Gergonne cevian AD.
$E = \texttt{Gergonne}[ABD]$.
$F = \texttt{Gergonne}[ADC]$.

Conclusion: $\texttt{concur}[AD, BE, CF]$.

References: This follows from the previous property.

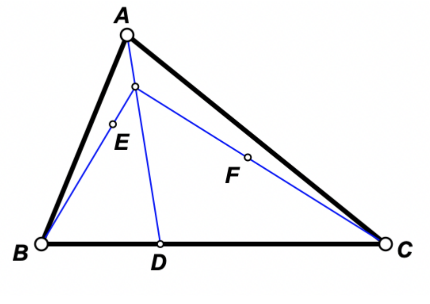


Property 3.3.3.

Start with:
Triangle ABC with Gergonne cevian AD.
$E = \texttt{Nagel}[ABD]$.
$F = \texttt{Nagel}[ADC]$.

Conclusion: $\texttt{concur}[AD, BE, CF]$.

References: [39]

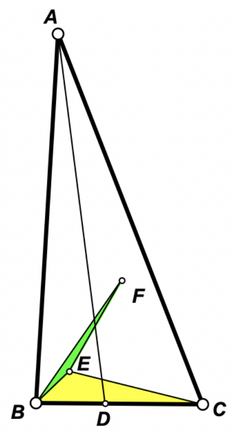


Property 3.3.4.

Start with:
Triangle ABC with Gergonne cevian AD.
$\triangle ABC$ satisfies $b + c = ka$.
$E = \texttt{incenter}[ABD]$.
$F = \texttt{centroid}[ADC]$.

Conclusion: $\dfrac{[BEF]}{[BCE]} = \dfrac{k-3}{6}$.

References: [54]*

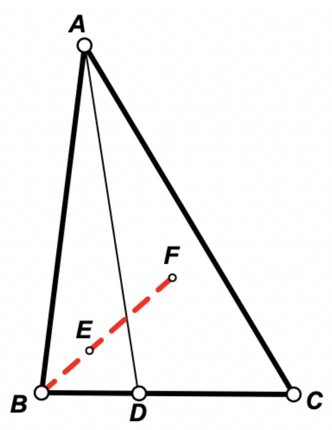


Property 3.3.5.

Start with:
Triangle ABC with Gergonne cevian AD.
$\triangle ABC$ satisfies $b + c = 3a$.
$E = \texttt{incenter}[ABD]$.
$F = \texttt{centroid}[ADC]$.

Conclusion: $\texttt{colline}[B, E, F]$.

References: Special case of $k = 3$ in previous property.

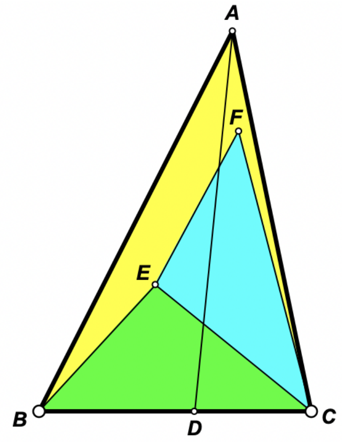


Property 3.3.6.

Start with:
Triangle ABC with Gergonne cevian AD.
$\triangle ABC$ satisfies $b+c=3a$.
$E = \texttt{centroid}[ABD]$.
$F = \texttt{Nagel}[ADC]$.

Conclusion: Three colored regions have the same area.

References: [55]*

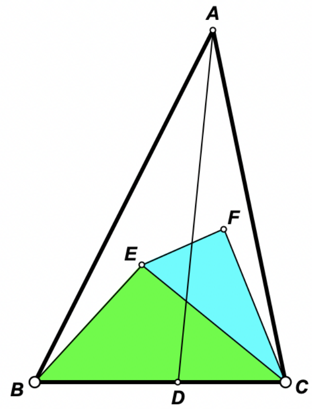


Property 3.3.7.

Start with:
Triangle ABC with Gergonne cevian AD.
$\triangle ABC$ satisfies $b+c=3a$.
$E = \texttt{centroid}[ABD]$.
$F = \texttt{Spieker}[ADC]$.

Conclusion: $[BCE] = 2[CFE]$.

References: [57]

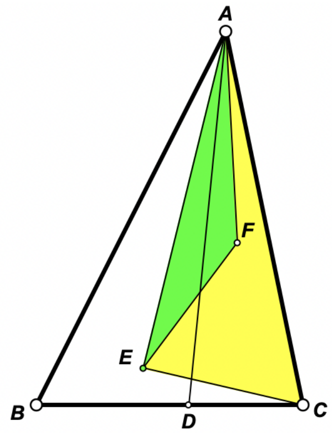


Property 3.3.8.

Start with:
Triangle ABC with Gergonne cevian AD.
$\triangle ABC$ satisfies $b+c=3a$.
$E = \texttt{Gergonne}[ABD]$.
$F = \texttt{Spieker}[ADC]$.

Conclusion: $3[AEC] = 8[AEF]$.

References: [56]*



4. A Triangle with Gergonne and Nagel Cevians

In this section, we report on properties found for a starting figure consisting of a triangle with two Gergonne cevians or a triangle with a Gergonne and a Nagel cevian.

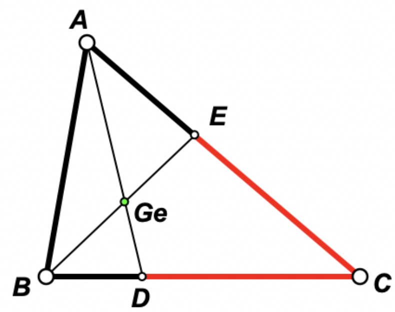
4.1. **Two Gergonne Cevians.**

Property 4.1.1.

Start with: Triangle ABC.
AD is a Gergonne cevian.
BE is a Gergonne cevian.

Conclusion: $CD = CE$.

References: This follows from the Trace Properties.

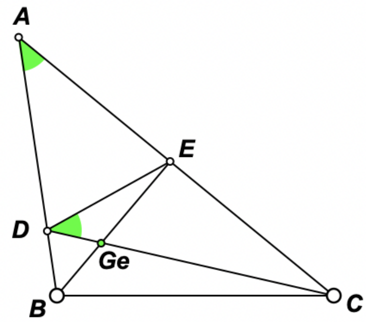


Property 4.1.2.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $c = 2(b - a)$.
BE is a Gergonne cevian.
CD is a Gergonne cevian.

Conclusion: $\angle CDE = \angle BAC$.

References: [5]

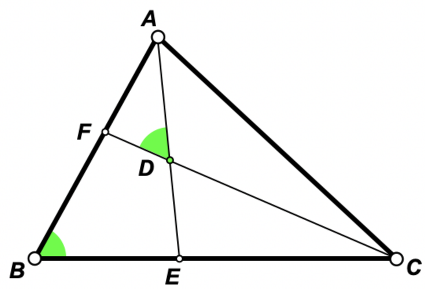


Property 4.1.3.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $b(s - b) = 2(s - c)(s - a)$.
AE is a Gergonne cevian.
CF is a Gergonne cevian.

Conclusion: $\angle ABC = \angle ADF$.

References: [15]

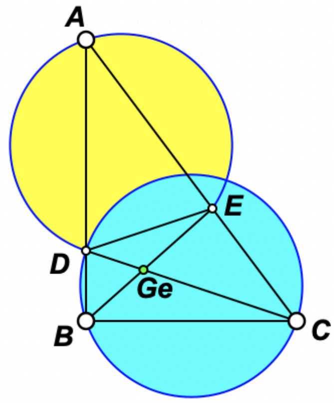


Property 4.1.4.

Start with: Right triangle ABC.
$\triangle ABC$ satisfies $a = 3(b - c)$.
BE is a Gergonne cevian.
CD is a Gergonne cevian.

Conclusion: $\odot BCD \cong \odot ADE$.

References: [81]

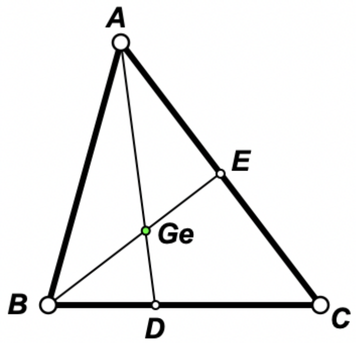


Property 4.1.5.

Start with: Triangle ABC.
AD is a Gergonne cevian.
BE is a Gergonne cevian.

Conclusion: $\dfrac{[ADC]}{[BCE]} = \dfrac{b}{a}$.

References: [22]



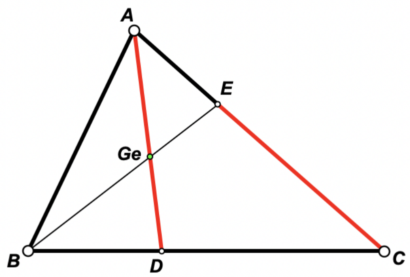
Property 4.1.6.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 16 : 15 : 11$.
AD is a Gergonne cevian.
BE is a Gergonne cevian.

Conclusion: $AD = CE$.

Note: This result is also true whenever $4s^3 + 3abc + a^2b = 4s(ab + bc + ca)$.

References: [66]



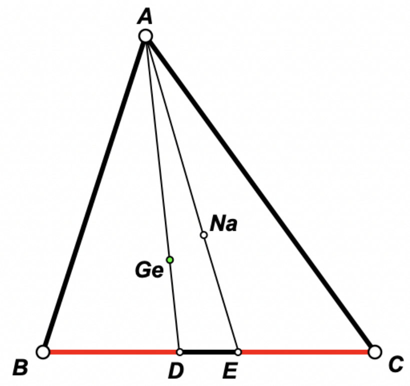
4.2. A Gergonne Cevian and a Nagel Cevian.

Property 4.2.1. Isotomic Conjugate

Start with: Triangle ABC.
AD is a Gergonne cevian.
AE is a Nagel cevian.

Conclusion: $BD = CE$.

References: [10, p. 184]

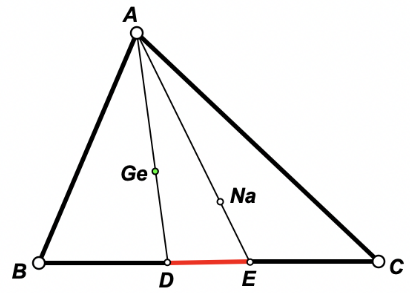


Property 4.2.2.

Start with: Triangle ABC.
AD is a Gergonne cevian.
AE is a Nagel cevian.

Conclusion: $|AB - AC| = DE$.
Triangles ABC and ADE have the same centroid.

References: This follows from the Trace Properties.

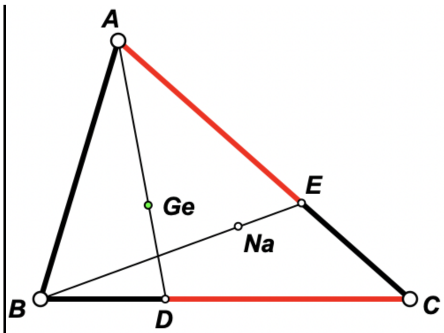


Property 4.2.3.

Start with: Triangle ABC.
AD is a Gergonne cevian.
BE is a Nagel cevian.

Conclusion: $AE = CD$.

References: Follows from Trace Properties

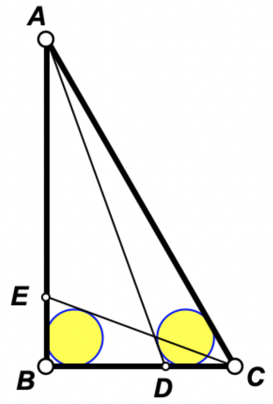


Property 4.2.4.

Start with: Right triangle ABC.
$\triangle ABC$ satisfies $\angle A = 30°$ and $\angle C = 60°$.
AD is a Gergonne cevian.
CE is a Nagel cevian.

Conclusion: $r(ADC) = r(BCE)$.

References: [24]

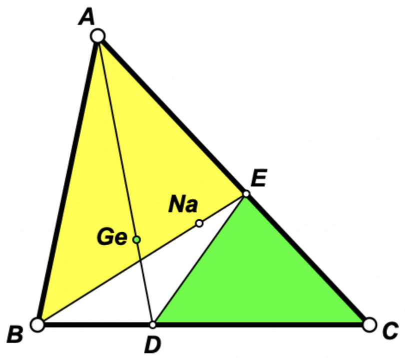


Property 4.2.5.

Start with: Triangle ABC.
AD is a Gergonne cevian.
BE is a Nagel cevian.

Conclusion: $\dfrac{[ABE]}{[CED]} = \dfrac{2a}{b+c-a}$.

References: [4]

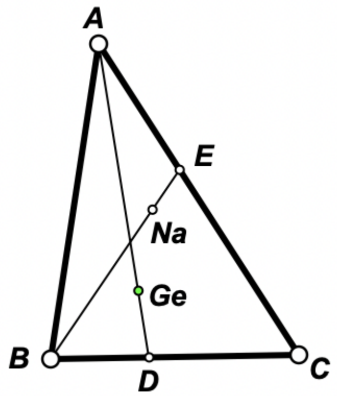


Property 4.2.6.

Start with: Triangle ABC.
AD is a Gergonne cevian.
BE is a Nagel cevian.

Conclusion: $\dfrac{[ADC]}{[ABE]} = \dfrac{b}{a}$.

References: [20]

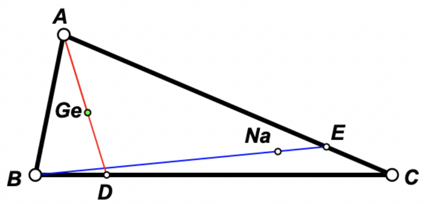


Property 4.2.7.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 5 : 5 : 2$.
AD is a Gergonne cevian.
BE is a Nagel cevian.

Conclusion: $BE = 2AD$ and $BN_a = 3AG_e$.

References: [62]



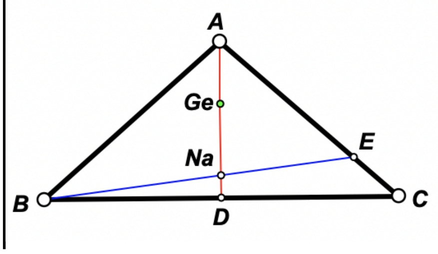
Property 4.2.8.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 3 : 2 : 2$.
AD is a Gergonne cevian.
BE is a Nagel cevian.

Conclusion: $BE = 2AD$.

Note: As in any triangle with $AB = AC$, A, G_e, and N_a are collinear.

References: [63]



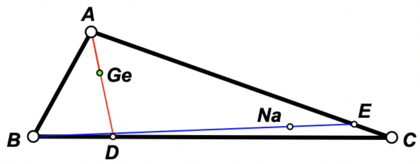
Property 4.2.9.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 9 : 8 : 3$.
AD is a Gergonne cevian.
BE is a Nagel cevian.

Conclusion: $BE = 3AD$.

Note: $BE = 3AD$ is also true for triangles where $a : b : c = \sqrt{41} - 3 : 2 : 2$.

References: [64]

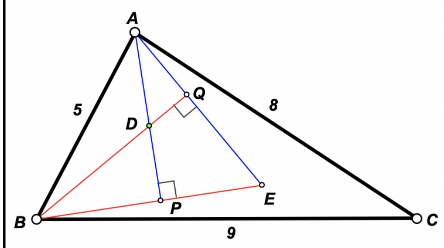


Property 4.2.10.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 9 : 8 : 5$.
$D =$ `Gergonne`$[ABC]$.
$E =$ `Nagel`$[ABC]$.

Conclusion: $AP \perp BE$ and $BQ \perp AE$.

References: [65]

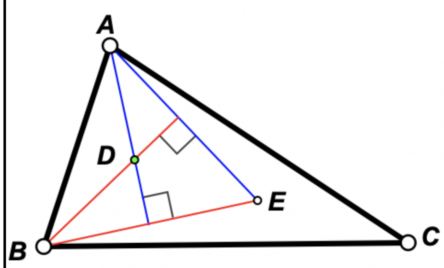


Property 4.2.11.

Start with: Triangle ABC.
$D =$ `Gergonne`$[ABC]$.
$E =$ `Nagel`$[ABC]$.

Conclusion: $AD \perp BE$ if and only if $BD \perp AE$.

References: [86]



5. A Triangle with a Cevian

In this section, we report on properties found for a starting figure that consists of a triangle ABC and one of its cevians. The cevian divides $\triangle ABC$ into two smaller triangles. A Gergonne point is constructed in one of these triangles or in the original triangle.

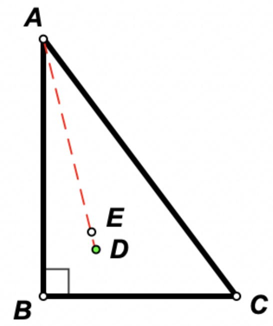
5.1. Gergonne Point in Original Triangle.

Property 5.1.1.

Start with: Right triangle ABC with cevian through the nine-point center.
$\triangle ABC$ satisfies $c = 2(b - a)$ and $\angle B = 90°$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{ninepointcenter}[ABC]$.

Conclusion:
(a) $\texttt{colline}[A, E, D]$.
(b) $\dfrac{AE}{ED} = \dfrac{3(2a + b)}{9b - 14a}$.

References: [77]

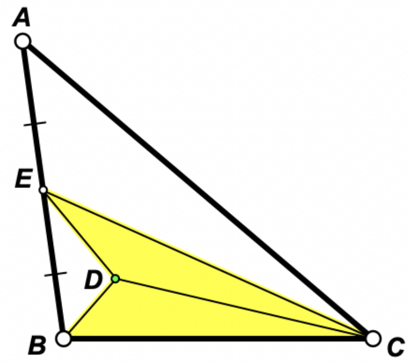


Property 5.1.2.

Start with: Triangle ABC with median.
$\triangle ABC$ satisfies $c = 2(b - a)$
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{midpoint}[AB]$.

Conclusion: $[BCD] = [CED]$.

References: [2]*

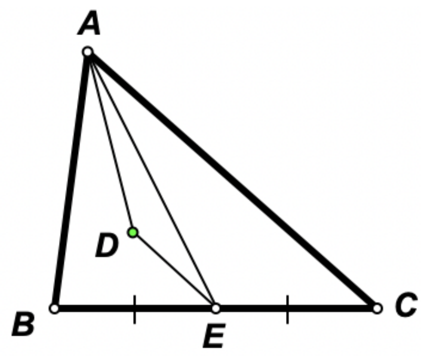


Property 5.1.3.

Start with: Triangle ABC with median.
$\triangle ABC$ satisfies $b + c = 2a$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{midpoint}[BC]$.

Conclusion:
$(AD)^2 + (AE)^2 = (DE)^2 + (BC)^2$.

References: [33]*



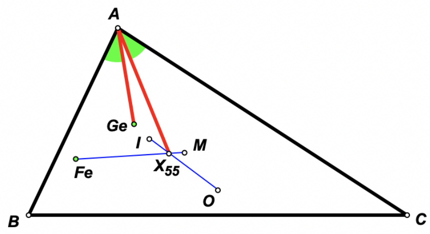
Property 5.1.4. Isogonal Conjugate

Start with:
Triangle ABC with an X_{55}-cevian.
$G_e = \texttt{Gergonne}[ABC]$.

Conclusion: $\angle BAG_e = \angle CAX_{55}$

Note: I is the incenter, O is the circumcenter, M is the centroid, and F_e is the Feuerbach point of $\triangle ABC$.

References: [13]



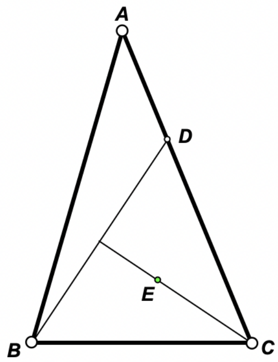
5.2. **Gergonne Point in One of the Smaller Triangles.**

Property 5.2.1.

Start with:
Triangle ABC with Nagel cevian BD.
$\triangle ABC$ satisfies $b + c = 3a$.
$E = \texttt{Gergonne}[BCD]$.

Conclusion: $CE \perp BD$.

References: [58]

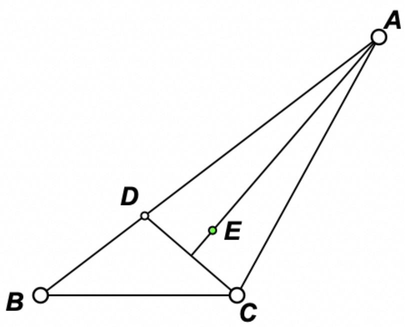


Property 5.2.2.

Start with:
Triangle ABC with symmedian CD.
$\triangle ABC$ satisfies $a^2 + b^2 = bc$.
$E = \texttt{Gergonne}[ADC]$.

Conclusion: $AE \perp CD$.

References: [80]

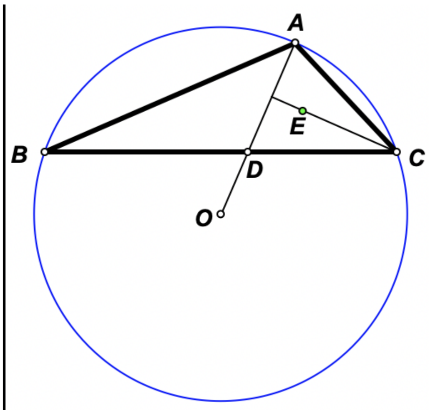


Property 5.2.3.

Start with:
Triangle ABC with X_3-cevian AD.
$\triangle ABC$ satisfies $\angle C = 2\angle B$.
$E = \texttt{Gergonne}[ADC]$.

Conclusion: $CE \perp AD$.

References: [59]

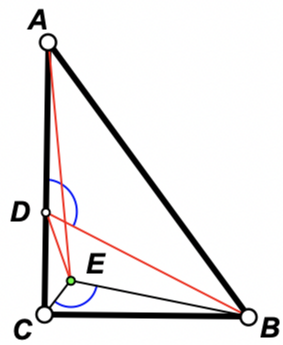


Property 5.2.4.

Start with:
Right triangle ABC with angle bisector BD.
$\triangle ABC$ satisfies $a : b : c = 3 : 4 : 5$.
$E = \texttt{Gergonne}[BCD]$.

Conclusion:
(a) $\angle BEC = \angle ADB$.
(b) $(DE)^2 + (BD)^2 = (AE)^2$.*

References: [40]

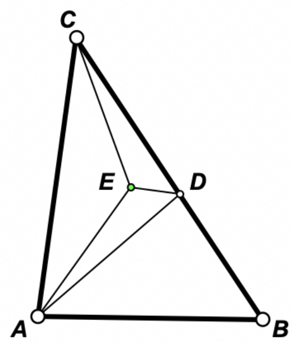


Property 5.2.5.

Start with:
Triangle ABC with angle bisector AD.
$\triangle ABC$ satisfies $a : b : c = 6 : 5 : 4$.
$E = \texttt{Gergonne}[ACD]$.

Conclusion:
(a) $\angle AEC$ and $\angle ABC$ are supplementary.*
(b) $(BE)^2 + (AC)^2 = (BC)^2$.*
(c) $E = \texttt{Nagel}[ABC]$.

References: [41]

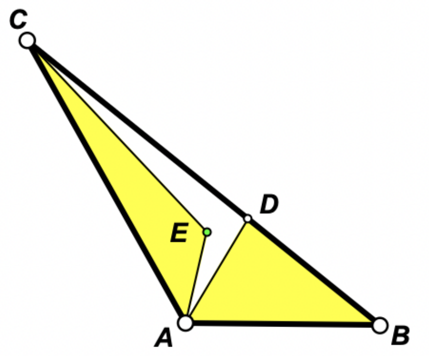


Property 5.2.6.

Start with:
Triangle ABC with angle bisector AD.
$\triangle ABC$ satisfies $a : b : c = 7 : 5 : 3$.
$E = \texttt{Gergonne}[ACD]$.

Conclusion: $[ABD] = [AEC]$.

References: [43]

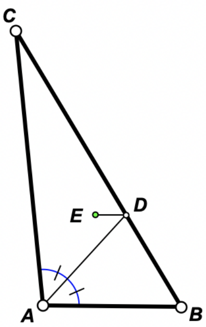


Property 5.2.7.

Start with:
Triangle ABC with angle bisector AD.
$\triangle ABC$ satisfies $a : b : c = 7 : 6 : 3$.
$E = \texttt{Gergonne}[ACD]$.

Conclusion:
(a) $AB \parallel DE$. (b) $BD + DE = AB$.
(c) $AC = 9DE$. (d) $CD = 7DE$.
(e) $CD + DE = 2AD$. (f) $AD = 4DE$.

References: [42]*

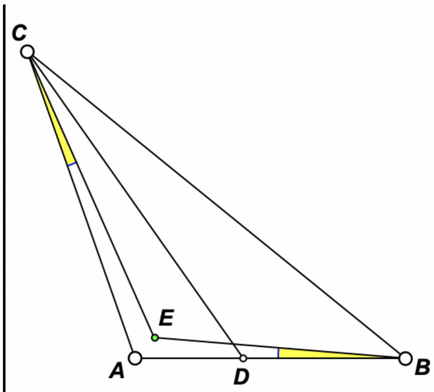


Property 5.2.8.

Start with:
Triangle ABC with angle bisector CD.
$\triangle ABC$ satisfies $a : b : c = 9 : 6 : 5$.
$E = \texttt{Gergonne}[ACD]$.

Conclusion: $\angle ABE = \angle ACE$.

References: [44]

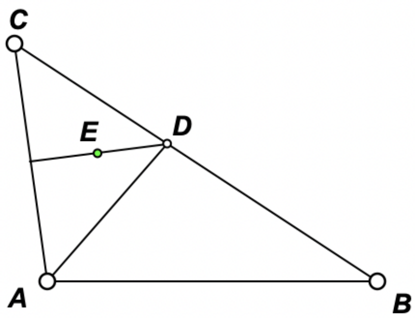


Property 5.2.9.

Start with:
Triangle ABC with angle bisector AD.
$\triangle ABC$ satisfies $a^2 = c(b + c)$.
$E = \texttt{Gergonne}[ADC]$.

Conclusion: DE is the perpendicular bisector of AC.

References: [41]



6. Lines Through the Gergonne Point

The line segment joining two points on the perimeter of a triangle is called a *chord*. A chord parallel to a side of the triangle is called a *parachord*. This is also known as a *parallelian* in the literature. The parachord through the Gergonne point of a triangle is called a *Gergonne parachord* (Figure 1, left). The line segment from the Gergonne point to the point where the Gergonne parachord meets a side of the triangle is called a Gergonne pararadius (Figure 1, right).

6.1. Properties of a Pararadius.

In this section, we report on properties found for a starting figure consisting of a triangle and a Gergonne pararadius.

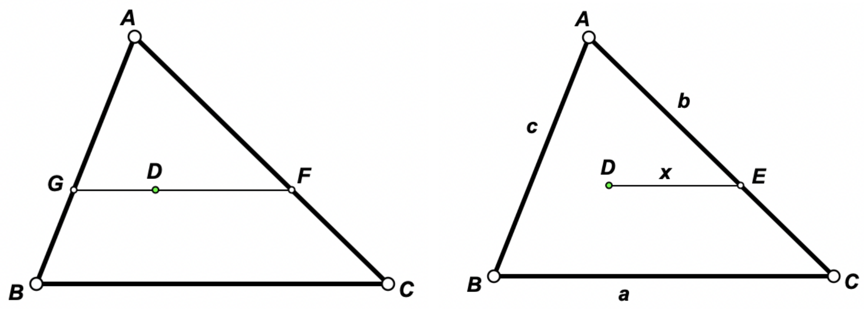


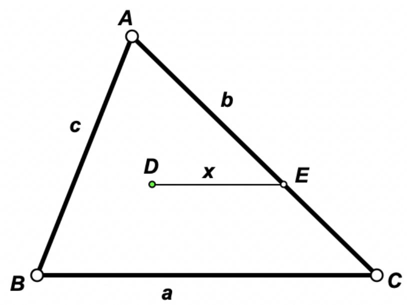
Figure 1. Gergonne Chord and Gergonne Pararadius

Property 6.1.1. **(Pararadius Formula)**

Start with:
Triangle ABC with Gergonne pararadius.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{parallel}[D, BC] \cap AC$.

Conclusion:
$$x = \frac{-a(a+b-c)(b+c-a)}{a^2+b^2+c^2-2ab-2bc-2ca}.$$

References: See supplementary notebook.

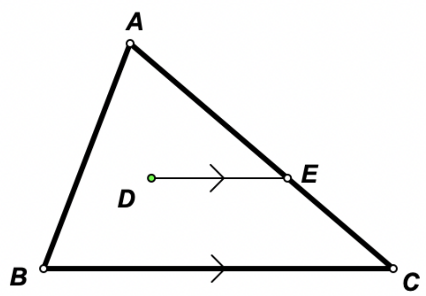


Property 6.1.2. **(Parallel Ratio)**

Start with:
Triangle ABC with Gergonne pararadius.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{parallel}[D, BC] \cap AC$.

Conclusion: $\dfrac{AE}{DE} = \dfrac{b}{s-c}$.

References: See supplementary notebook.



Property 6.1.3.

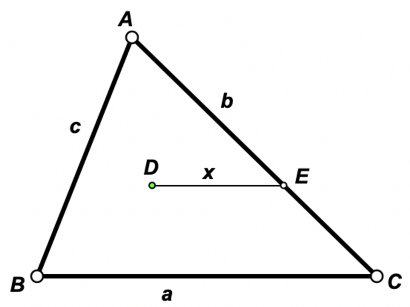
Start with:
Triangle ABC with Gergonne pararadius.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{parallel}[D, BC] \cap AC$.

Conclusion:

$$AE = \frac{2ab(a-b-c)}{a^2+b^2+c^2-2ab-2bc-2ca}.$$

$$CE = \frac{b(b^2+c^2-a^2-2bc)}{a^2+b^2+c^2-2ab-2bc-2ca}.$$

References: See supplementary notebook.

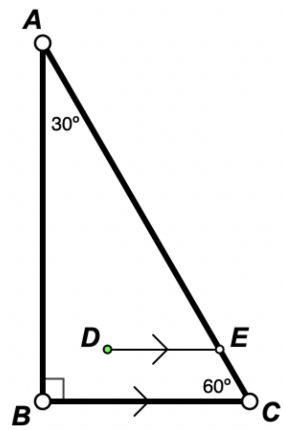


Property 6.1.4.

Start with:
Triangle ABC with Gergonne pararadius.
$\triangle ABC$ satisfies $\angle B = 90°$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{parallel}[D, BC] \cap AC$.

Conclusion: $BC + AE = \left(1 + \frac{2b}{a}\right) DE$.

References: [84]. See also [26].

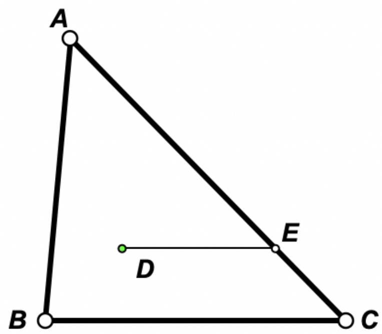


Property 6.1.5.

Start with:
Triangle ABC with Gergonne pararadius.
$\triangle ABC$ satisfies $c = 3(b-a)$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{parallel}[D, BC] \cap AC$.

Conclusion: $\frac{1}{AE} + \frac{1}{DE} = \frac{1}{CE}$.

References: [78]

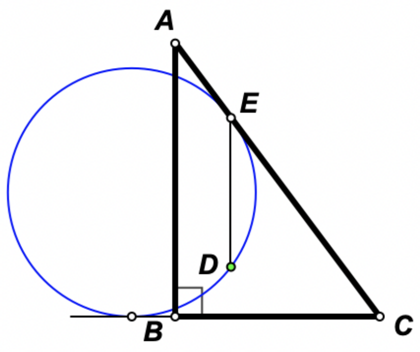


Property 6.1.6.

Start with:
Triangle ABC with Gergonne pararadius.
$\triangle ABC$ satisfies $a : b : c = 3 : 5 : 4$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{parallel}[D, AB] \cap AC$.

Conclusion: The circle through D touching AC at E is tangent to BC.

References: [35]

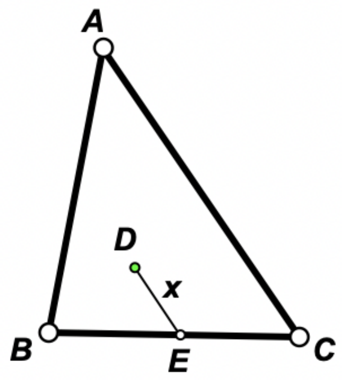


Property 6.1.7.

Start with:
Triangle ABC with Gergonne pararadius.
$\triangle ABC$ satisfies $c = (2 - \sqrt{5})a + b$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{parallel}[D, AC] \cap BC$.

Conclusion: $x = \dfrac{2b(c-b)}{a-3b+c}$.

References: [36] and [76]



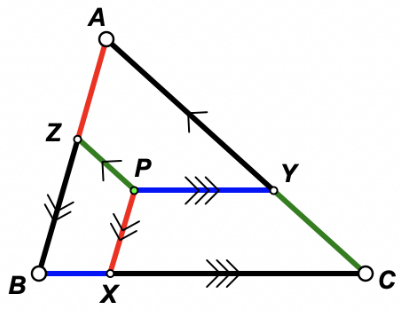
Property 6.1.8.

Start with:
Triangle ABC with three Gergonne pararadii.
$P = \texttt{Gergonne}[ABC]$.
$X = \texttt{parallel}[P, AB] \cap BC$.
$Y = \texttt{parallel}[P, BC] \cap CA$.
$Z = \texttt{parallel}[P, CA] \cap AB$.

Conclusion:

$$\frac{1}{AZ} + \frac{1}{PX} = \frac{1}{BX} + \frac{1}{PY} = \frac{1}{CY} + \frac{1}{PZ}.$$

References: [47]



6.2. Properties of a Parachord.

In this section, we report on properties found for a starting figure consisting of a triangle and a Gergonne parachord.

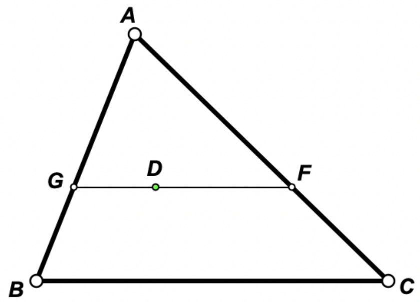
Property 6.2.1. **(Parachord Formula)**

Start with:
Triangle ABC with Gergonne parachord.
$D = \texttt{Gergonne}[ABC]$.
$F = \texttt{parallel}[D, BC] \cap AC$.
$G = \texttt{parallel}[D, BC] \cap AB$.

Conclusion:

$$FG = \frac{2a^2(a-b-c)}{a^2+b^2+c^2-2ab-2bc-2ca}.$$

References: This follows from the Pararadius Formula.

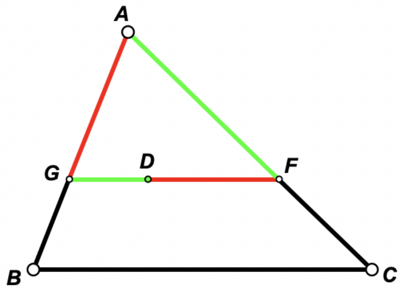


Property 6.2.2. **(Parachord Perimeter)**

Start with:
Triangle ABC with Gergonne parachord.
$D = \texttt{Gergonne}[ABC]$.
$F = \texttt{parallel}[D, BC] \cap AC$.
$G = \texttt{parallel}[D, BC] \cap AB$.

Conclusion: $AG + DF = AF + DG$.

References: This follows from the Split Perimeter Property.



6.3. Properties of Gergonne Chords.

In this section, we report on properties found for a starting figure consisting of a triangle and a Gergonne chord.

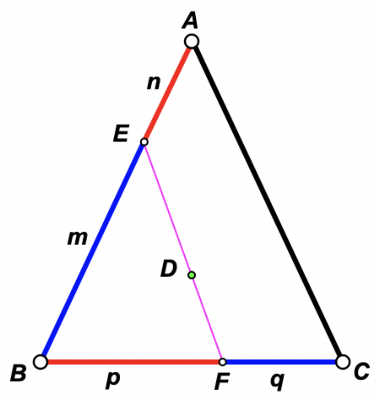
Property 6.3.1.

Start with: Isosceles triangle ABC.
EF is a Gergonne chord.
$\triangle ABC$ satisfies $b = c$.
$D = \texttt{Gergonne}[ABC]$.
$F = ED \cap BC$.

Conclusion:

$m(2m + 2n - p - q)(p - q) = np(p + q)$.

References: [49]*



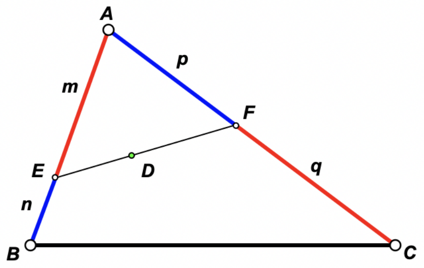
Property 6.3.2.

Start with: Triangle ABC.
EF is a Gergonne chord.
$D = \texttt{Gergonne}[ABC]$.
$F = ED \cap AC$.

Conclusion:

$$\frac{s-a}{s-b} \cdot \frac{n}{m} + \frac{s-a}{s-c} \cdot \frac{q}{p} = 1.$$

References: [73, p. 46]



6.4. Perpendiculars From the Gergonne Point.

In this section, we report on properties found for a starting figure consisting of a triangle and one or more perpendiculars dropped from the Gergonne point to the sides of the triangle.

Property 6.4.1. (Apothem Length)

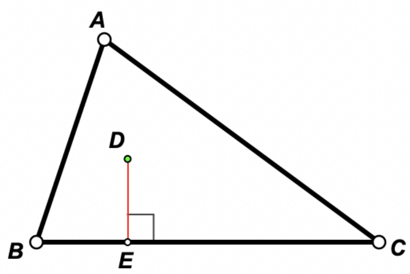
Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{perpendicular}[D, BC]$

Conclusion:

$$DE = \frac{8(s-b)(s-c)K}{a(2ab+2bc+2ca-a^2-b^2-c^2)},$$

where $K = \sqrt{s(s-a)(s-b)(s-c)}$.

References: This follows from the area formula.

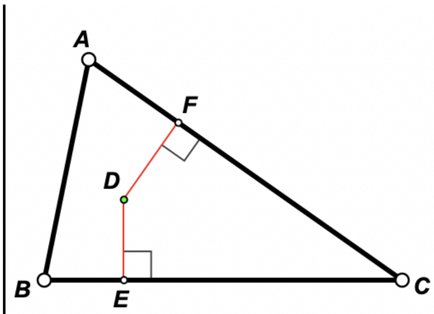


Property 6.4.2. (Trilinear Coordinates)

Start with: Triangle ABC.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{perpendicular}[D, BC]$
$F = \texttt{perpendicular}[D, AC]$

Conclusion: $\dfrac{DE}{DF} = \dfrac{b(c+a-b)}{a(b+c-a)}$.

References: [12]

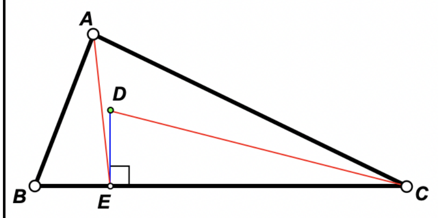


Property 6.4.3.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 16 : 15 : 7$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{perpendicular}[D, BC]$

Conclusion: $CD = 2AE$.

References: [67]

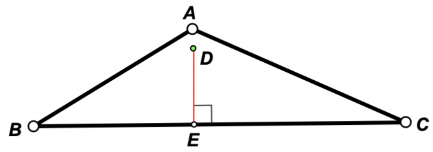


Property 6.4.4.

Start with: Triangle ABC.
$\triangle ABC$ satisfies $a : b : c = 8 : 5 : 4$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{perpendicular}[D, BC]$

Conclusion: $\frac{[BCD]}{[AED]} = 632$.

References: [60]

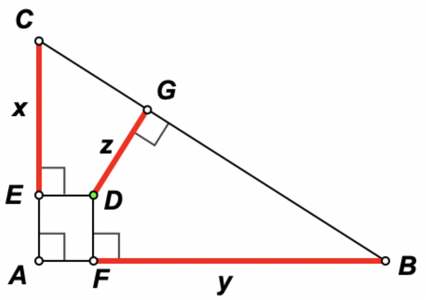


Property 6.4.5.

Start with: Right triangle ABC.
$\triangle ABC$ satisfies $\angle A = 90°$.
$D = \texttt{Gergonne}[ABC]$.
$E = \texttt{perpendicular}[D, AC]$
$F = \texttt{perpendicular}[D, AB]$
$G = \texttt{perpendicular}[D, BC]$

Conclusion: $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.

References: [61]



7. OTHER PROPERTIES DISCOVERED BY COMPUTER

Other properties of the Gergonne point (discovered by computer) were found by Dekov [6]. A typical result is: The Gergonne point of a triangle is the mittenpunkt of the orthic triangle of the intouch triangle.

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